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What does an observed quantum system reveal to its observer?

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Abstract. The evolution of a quantum system under observation becomes retarded or even impeded. We review this "quantum Zeno effect" in the light of the criticism that has been raised upon a previous attempt to demonstrate it, of later reexaminations of both the projection postulate and the significance of the observations, and of the results of a recent experiment on an *individual* cold atom. Here, the microstate of the quantum system gets unveiled with the observation, and the effect of measurement is no longer mixed up with dephasing the object's wave function by the reactive effect of the detection. A procedure is outlined that promises to provide, by observation, an upper limit for the delay of even an *exponential* decay.

PACS. 03.65.Bz Foundations, theory of measurement, miscellaneous theories (including Aharonov-Bohm effect, Bell inequalities, Berry's phase) – 32.80.-t Photon interactions with atoms – 42.50.Ct Quantum description of interaction of light and matter; related experiments

1 Introduction

An isolated quantum system found in one of its eigenstates of energy starts evolving as soon as it is perturbed: this system when supplemented by the perturbing entity no longer resides in one of its eigenstates, and it becomes time-dependent. This phenomenon seems close to the behaviour of a macroscopic system, say of a pair of coupled pendula, with its split normal modes. However, an attempt of measuring an observable of the quantum system brings to the fore a fundamental difference: here, one of the eigenvalues is the result of the measurement. Although we have become accustomed with this fundamental teaching of quantum mechanics, still alien seems the unavoidable conclusion that the system has suddenly turned into the eigenstate corresponding to the observed eigenvalue. This "state reduction" according to the projection postulate of von Neumann [1] and Lüders [2] is thought to accompany the observation of the system and constitutes, as it seems, a discontinuous break of the otherwise continuous quantum evolution. Repeated measurement, with same result, on a quantum system amounts to reiterated resettings of the evolving wave function. It brings about retardation or even impediment of the evolution [3,4], when the temporal separation of the detections is reduced, and the interaction more and more approaches continuous measurement. This theoretical consequence, *i.e.* quantum evolution impeded by measurement, or the "quantum Zeno effect" (QZE, [5]), is based on the likeliness of finding the system in a state *other* than the initial one varying in proportion with the square of lapsed time, $(\Delta t)^2$ [3,6–9]. With N quasi-instantaneous observations of the system equally distributed during time T and separated by T/N, that net probability varies as $N(\Delta t)^2 = T^2/N$, whose limit at $N \to \infty$ vanishes, and with it the dynamics of the system.

An attempt of demonstrating the QZE has involved a large ensemble of atomic particles well isolated from the environment, namely five thousand beryllium ions located in an ion trap [10]. Another experiment that made use of light in cascaded interferometers [11] has been shown explicable in classical terms [8]. The ion ensemble in the trap experiment [10] interacted, aside from the trapping field, only with two kinds of pulse of radiation: long microwave pulses that drove the ensemble's coherent quantum evolution on a hyperfine line, and a series of short light pulses superimposed for the excitation of light scattering on a neighbouring resonance line, that was assumed to represent a sequence of measurements of the system's internal energy state.

This experiment has yielded complete agreement with the predictions of quantum-mechanical calculations. Its analysis and the pertaining claims have, however, aroused criticism on various counts:

(i) the interpretation of the experimental findings in terms of projection postulate and state reduction that was said inappropriate [12,13];
(ii) the interpretation of these findings in terms of the quantum Zeno effect that was said inappropriate *because it did not require* the application of the projection postulate [14–16];
(iii) the recording only of the *net* probability for no transition, after a *series* of short light pulses applied to the

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ions, in spite of the application of every individual pulse being considered a "measurement" [17];
(iv) the results of the observations being considered not to demonstrate a non-local, negative-result effect [8];
(v) the use, for the quantum system, of a large ensemble that makes one identify the effect of the measurements with physically dephasing the wave function [18–23];
(vi) the demonstration of the perturbed evolution on a *coherent* dynamics, as opposed to spontaneous, exponential decay [24].

The review of Home and Whitaker [8] gives a very detailed account of many relevant problems and reevaluates the interpretations. It is the purpose of the present paper to review the positions characterized by the arguments listed above in the light of this and other reevaluations, and of a recent experiment on an *individual* atomic system [25]. We feel that this contribution to the debate is well justified since, after all, the QZE seems to characterize a non-local correlation of the quantum system with a macroscopic meter, as Bell's inequalities characterize the non-local correlation of two quantum systems [8].

2 Projection postulate and state reduction

A principal line of criticism of the experiment of Itano *et al.* [10] is concerned with the application of projection postulate and state reduction to the interpretation [12,13,19]: a particular degree of freedom – the weak resonance of an atomic two-level system being excited by microwave used as the drive – was said to have been inappropriately singled out. When the probe transition – the resonance line – were included in the model, as, *e.g.*, in a set of *three*-level Bloch equations, the mean evolution of the entire system is said derivable complete with situations of almost vanishing non-diagonal elements of the density matrix. There is no need to separately invoke projection and state reduction, which in fact are inferred not to happen [13].

The reasoning suggests that in fact the concept of state reduction is not required for the formal description of the dynamics of the driven quantum system extended by what has been named "quantum probe" [21], and of the relationship of driven resonance and probe resonance. However, the photo-electric detection of the scattered light still reduces the extended system into the "on" state, and failing to detect scattering reduces it into the "off" state. Reduction of the extended system is now achieved by the next link of the causal chain so far *not* included in the model, *i.e.* here by the photo-detection of the probe-light-excited fluorescence. Thus, this concept seems inevitable just at the borderline separating the modelled system (now the complete three-level system) and the rest of the world that includes the macroscopic measuring device. Moreover, the modelling of the multi-particle system based on the density matrix results in expectation values of the observable that correspond to "non-selective" measurements, *i.e.* observations that do not reveal the micro-state of the system. Such a type of observation, however, is inappropriate for the demonstration of the reactive effect of a *measurement* since this effect, in such an observation, completely agrees with physically dephasing the system's wave function [18–22]. This ambiguity of interpretation will be discussed in Section 4. A quantum system subject to selective measurements, *i.e.* observations linked with detailed information on the relevant observables of individual quantum objects that constitute the system under scrutiny, is suitably modelled by the quantum jump approach [26] also known as Monte Carlo wave function calculation [27]. Here, trajectories of the evolution of an individual quantum system, or of the sub-systems of a larger entity, are calculated whose weighted average will eventually reproduce the results of a corresponding calculation based on the density-matrix, *e.g.*, the solutions of Bloch equations. Using the quantum jump approach, Beige and Hegerfeldt [9] have shown that a "good measurement" as defined by perfect projection and state reduction is approximated to very high precision by the effect of a probe pulse on a resonant atom. This is so in the range of parameters defined by $A\Omega_{\rm d}/\Omega_{\rm p} \ll 1$ and $\Omega_{\rm p}/A \ll 1$, where $\Omega_{\rm d}$, $\Omega_{\rm p}$ are the Rabi frequencies of the drive and probe radiation, respectively, and A is the rate of spontaneous decay on the probe line. Related results have been derived from numerical simulations using Bloch equations [28]. The parameters of the experiment of Itano *et al.* [10] are well inside the above regime, such that "state reduction" remains a very good characterization of the effect of a probe-light pulse. Consequently, the objections on the ground of the use of this concept for the interpretation of the experimental findings seem unfounded.

On the other hand, Home and Whitaker have argued that not state reduction and "collapse" are prerequisite for the "paradoxical" aspects of the quantum evolution to show up, but rather the quadratic dependence on time of the probability for survival of the system in its initial state [8]. Thus, objections on the ground of *not* using the concept of collapse seem also irrelevant.

3 Measurement of the net rate of survival instead of the true rate

The concept of the evolution of a quantum system to be inhibited by reiterated measurement according to the proposal of Cook was based upon the measurable variation of the probability that *no* decay has been found throughout the interval $\delta = [0, T]$, where T is the overall time of the (driven) evolution [29]. Nakazato *et al.* [17] have pointed out that in the experiment on trapped Be ions [10] actually a different quantity has been recorded, namely the probability of finding no *net* decay at time T, while N interactions with the probe light, being considered measurements, have happened during δ. Although the results of these N "measurements" in principle could have been recorded, in fact they were not; instead, the state of the system was recorded *past* δ, *i.e.* at time T. The emergent overall results ignore processes that involve the excitation of some ion, time-correlated with deexcitation of any other ion. Moreover, such measurements, even when exerted upon an *individual* particle, could not help unravelling the true probability of *no* transition since it would include an unknown number of back-and-forth transitions that have happened on the driven line of this single quantum system but are left unrecorded. As a consequence, the probability for no *net* transition is at variance with the probability for no transition whatsoever. The latter, however, is used with both the definition [5] and the quantitative evaluation of the effect of measurement upon the evolution of a quantum system. Although both the two probabilities approach unity with increasing number N of probe pulses, they differ at finite N. This is, however, the typical situation encountered with an experimental proof. In contrast, a series of measurements, during the interval δ, whose results are being *individually recorded*, would lack that drawback and permit the determination of the probability that indeed no transition (decay or excitation) has happened during the entire time of interaction. The recent experiment that has in fact involved the registration of *all* the results of the N measurements of a sequence, exerted on a single ion [25], will be discussed in Section 5, and its possible extension to decay processes in Section 8.

4 Basis and signature of the quantum Zeno effect

Various types of observations have been suggested for the demonstration of the QZE both in model and experiment. The early proposals refer to the decay of an unstable quantum system [3–5]. Conventional models of such a decay inevitably require deviations from the exponential law at long and at short times [7,30,31], in particular an initial quadratic time dependence. However, these deviations have never been observed, and the question was raised of QZE to show up or not with a *strictly* exponential decay. Such a decay is a classical concept. Recently, however such a quantum model was constructed [7] on the expense of a conceptual drawback, namely the mean energy of the (position) eigenstates not being well-defined. Other models of the exponential decay have made use of the Wigner-Weisskopf approach that includes the physical interaction of the quantum object with an infinite reservoir [32]. These models feature unbounded spectra of energy eigenvalues and lack a lowest, stable, eigenstate [4,7]. Several recent proposals seem based on various types of such interaction; and they demonstrate even *advanced* evolution [33,34]. However, qualification for QZE *proper* (or QZ paradox [8]) seems questionable since the paradoxical aspects of QZ rely on the retardation of evolution in the *absence* of any reaction, on the quantum object, from the environment, namely as a consequence of non-local correlation of quantum object and meter. This situation may be unambiguously represented with the *coherent* evolution of a quantum system which consequently has become widely accepted as a model for the demonstration of the QZE [29].

This point of view has been elaborated in the comprehensive review of Home and Whitaker [8]. These authors stress the proximity of *this* QZE, based on correlation of quantum object and macroscopic detector, and Bell's theory, that involves the non-local correlation of two quantum objects. They suggest, for a consequent terminology, that "QZE" should characterize non-local negative result measurements on a microscopic system. Such a kind of observation would certainly suffice for discriminating the effect of correlation against back action and deserve the characterization as QZE: the information provided to the observer by the measurements is *not* generated *by local interaction.* However, it seems to us that the latter criterium is obeyed even by two more classes of measurements that are *not* of the negative-result type:

(1) measurements free of back action – the so-called "quantum non-demolition measurements" (QND) [21] – that may in fact give rise to positive results,
(2) measurements whose back action *demonstrably* cannot account for the surmised retarding effect, *e.g.*, because it is to small.

From a logical point of view there is no good reason to exclude these classes of measurements from the general type that generates the *true* QZE (or, "QZ paradox"), that might well be considered constituting a *necessary* criterium.

A factual demonstration according to criterium 2 may be very cumbersome, or even practically impossible, with a quantum system of many degrees of freedom. As for the analysis of the experiment of Itano *et al.* [10], the evaluation of reference [8] is still assumed to hold since that experiment fails to meet even the more general criteria 1 and 2. The situation is quite different with an individual quantum system when picking, for the observable, the simplest degree of freedom, a two-level system equivalent to a spin. Here, the back action on the quantum object to be scrutinized may turn out restricted to the small variation of a simple wave function, *i.e.*, of a modulus and/or a phase. In such a system, modification of the modulus goes along with a variation of energy. Such a variation, in particular any dissipation, prevents a measurement from being "good", but it may be excluded by the design of the experiment. A phase perturbation is harder to avoid. This problem will be addressed in the next two sections. A recent experiment on a *single* spatially confined ion will be analysed, and we shall show that part of the results obeys criterium 2, while another part obeys criterium 1 *and* even the sufficient criterium of reference [8].

5 Dephasing of the wave function of the observed system

Several authors have noticed that the effect of even a "good" measurement *on an ensemble* nonetheless completely agrees with phase diffusion of the system's wave function by the action of the environment and/or the measuring device ("dephasing") [18–23]. Spiller [22] as well as Alter and Yamamoto [23] have pointed out that the effect of measurement is discriminated from dephasing *only*

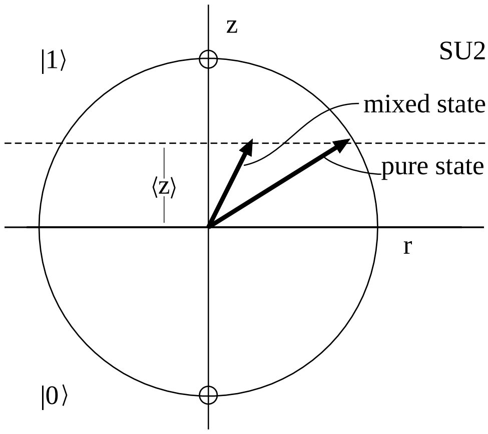

Fig. 1. The configuration space of an ensemble of two-level atoms fills surface *and* interior of the Bloch sphere. A measurement yields an expectation value $\langle \mathbf{z} \rangle$ with $0 \leq \langle \mathbf{z} \rangle \leq 1$. The configuration space of an individual such atom is restricted to the surface. A measurement of internal energy yields the eigenvalue 0 or 1, and a series of measurements, after equal preparation, reveals the micro-state that is a pure one.

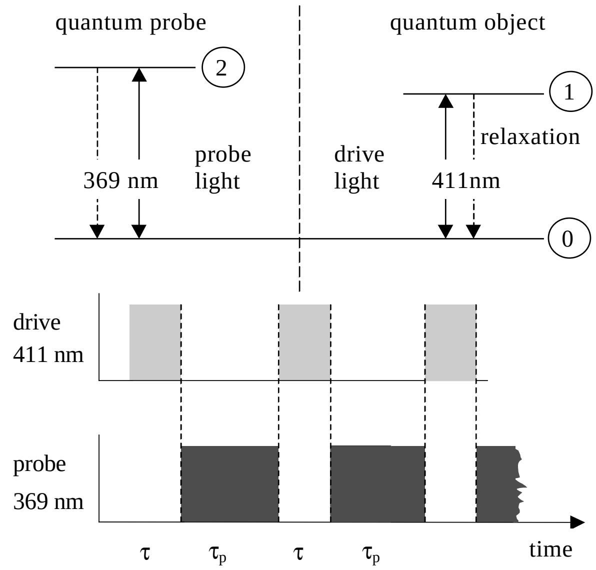

Fig. 2. Level scheme of an ion (Yb^+; level 0: $S_{1/2}$, 1: $D_{5/2}$, 2: $P_{1/2}$) interacting with monochromatic drive light resonant with $E2$ transition, and with probe light that excites light scattering on a resonance transition.

when the quantum object of the measurement is an *individual* quantum system. This is so since here indeed the micro-state of the measured observable becomes accessible to the observer, in contrast with a global measurement on an ensemble. As an example, we again consider the conceptionally simplest quantum system, a two-level atom, isomorphic with a spin 1/2 whose symmetry is SU2 [35]. The configuration space of such a system extends over the surface of the unit sphere whose poles correspond to the two energy eigenstates $\pm\, \hbar\omega_0/2$ (Fig. 1). All the other locations on this surface represent superposition states and exhibit a moment. Aside from the relative phase of the two eigenstates, this state may be determined from repeated measurements of energy upon identically preparing the quantum system, *i.e.*, of the projection of the state onto the z-axis, $|0\rangle - |1\rangle$. This is in contrast with an ensemble of such spins which requires one measurement only and represents, in general, a mixture of states whose state vector may terminate anywhere in the interior of the unit sphere: the reconstruction of the micro-state from the measurement is impossible as a consequence of incomplete knowledge, more specific: of ignoring the modulus of the moment that indicates the temporal correlations of the two eigenstates. Analytic [21] as well as numerical calculations [23] have proven that the evolutive dynamics of an ensemble under reiterated observation remains indistinguishable from an evolution subject to phase relaxation. Consequently, an unequivocal demonstration of the quantum evolution to be impeded or retarded by repeated *measurements* requires one to pick an individual quantum object as the system to be kept under observation.

6 Evidence against dephasing

Recently, an experiment on a single trapped and cooled ion has been reported [25]: the evolution of the ion was deduced from the statistics of sequences of *equal results* of measurements, each measurement consisting of driving and probing the ion on the respective neighbouring resonances. For this purpose, a single $^{172}Yb^+$ ion was spatially confined in a miniaturized electrodynamic trap in ultra-high vacuum [36]. The ion was laser-cooled and adjusted to the node of the radio-frequency trapping field such that it could be considered, in good approximation, as located in field-free space. For time intervals $\tau = 2$ ms, the $E2$ transition $S_{1/2}$–$D_{5/2}$ was excited by almost monochromatic blue light (1-s bandwidth less than 500 Hz) generated from frequency-doubling the 822-nm output of a diode laser, so that driving the ion was phase-coherent during τ (Fig. 2). These pulses alternated with 10-ms pulses of 369-nm probe light, generated by frequency-doubling the light of a dye laser. This probe light excited resonance scattering by the ion at a rate of 10^8 photons per second (of which some 10^4 were detected by photon counting) appeared *only* when the ion resided in the $S_{1/2}$ ground state and was susceptible to excitation of its dipole moment on the resonance line considered the quantum probe. Lacking light scattering was considered the signature of the ion being found in its *metastable* D state [37,38]. Trajectories of 500 measurements have been recorded, each consisting of a drive pulse, and a probe pulse with simultaneous detection. Pairs of measurements the first of which yields the ion in the ground state, and the second one in the metastable state, signal an act of excitation by the drive light. From the number of such pairs observed in a trajectory, the probability of excitation was derived. Trajectories recorded at the driving light being stepwise scanned across the pertaining line allow one to generate an absorption spectrum of the ion's $E2$ resonance. A corresponding spectrum calculated from a numerical solution of Bloch's equations is shown in Figure 3. The

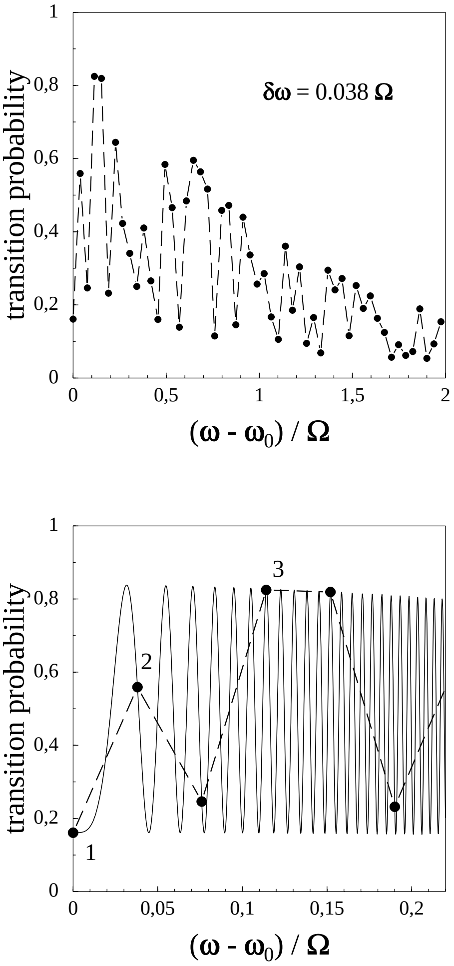

Fig. 3. Excitation probability $p_{01} = 1 - p_0$ on coherently driven transition 0–1, equivalent to equation (2), but simulated from numerical evaluation of Bloch equations. Top: the spectrum demonstrates the generation of the observed data of Figure 3, reference [25] (Rabi frequency $\Omega = 500$ kHz$\times 2\pi$). Bottom: expansion of the scale of detuning reveals Rabi nutation and the stroboscopic sampling with step size $\delta\omega$. The marks 1, 2, and 3 indicate transition probabilities that correspond to trajectories of data evaluated in Figure 4.

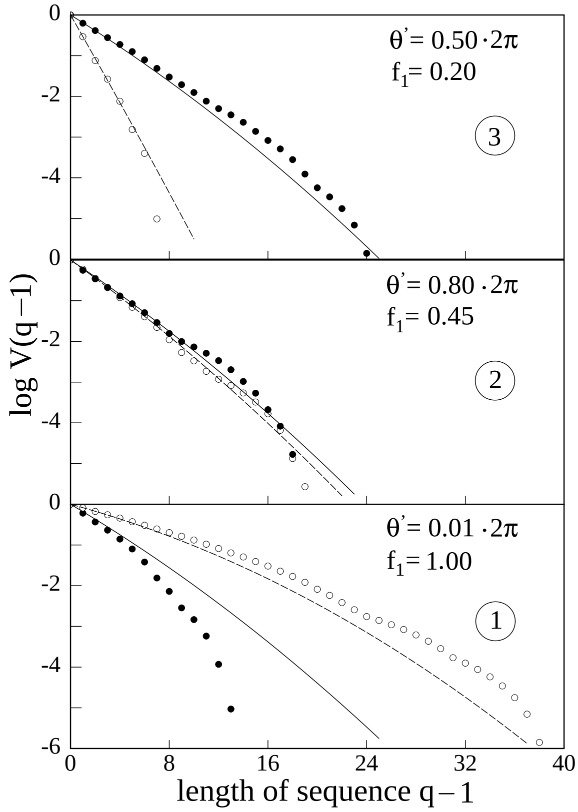

Fig. 4. Probabilities $U(q)/U(1)$ of uninterrupted sequences of q "on" results (white dots) and "off" results (black dots). The lines show the distributions of probabilities $V(q-1)$ for the ion's evolution on its drive transition, according to equations (2, 3). θ' and f_1 from fit; values $f_1 < 1$ indicate redistribution, over sublevels, by cycles of spontaneous decay and reexcitation. From reference [25].

probability of excitation oscillates according to the angle of optical nutation $\theta(t) = \sqrt{\Omega^2 + \Delta^2}\ t$ that depends on the Rabi frequency Ω and the detuning of the driving light, $\Delta = \omega - \omega_0$, where ω, ω_0 are the light and $E2$ resonance frequencies, respectively. The envelope of this absorption line is characterized by the Rabi nutation frequency, whereas pulse length of the drive and the additional broadening by the rate of dephasing, γ, determine the contrast of the modulation. Marked are those data points in the spectrum whose trajectories of measurement have been subjected to statistical evaluation. The recorded version of the excitation spectrum [25] agrees with Figure 3 and shows indeed the probability of excitation being Rabi modulated. This agreement reveals the interaction of driving light and ion to have been *coherent* in the experiment.

For the statistical evaluation of the results, the ion was assumed to occupy its ground state, when probe light was about to be scattered. Then, another such "on" event takes place with probability $p_0 = \cos^2(\Omega\tau/2)$, provided that the drive light is tuned to resonance ($\Delta = 0$). With the ion in its metastable state, the probability for another "off" result is the same, $p_1 = p_0 = p$, as long as relaxation is neglected. Finding a series of q equal results right after each other has the conditional probability

$$U(q) = U(1)V(q-1)\ , \tag{1}$$

where $V(q) = p^q = \cos^{2q}(\Omega\tau/2)$ is the conditional probability of the ion remaining in its original eigenstate under q attempts of coherent excitation or deexcitation. In contrast, completely preserved correlations − equivalent to the absence of "state reduction" or "collapse" − would require $V_{\mathrm{coh}}(q) = \cos^2(q\Omega\tau/2)$.

Relaxation modifies the probabilities p_i. An analytic solution of Bloch's equations on resonance [39] yields, for $\theta \gg \pi$, when dispersive interaction is negligible,

$$p_i = 1 - f_i B_i (1 - \mathrm{e}^{-(a+b)} \cos\theta), \tag{2}$$

where $B_0 = (\Omega^2/2)/(\Omega^2 + \Gamma\gamma)$, $B_1 = 1 - B_0$, $2a = \gamma_{\rm ph}\tau + (\Gamma/2)\tau$, $2b = \Gamma\tau$, $\theta^2 = (\Omega\tau)^2 - (a-b)^2$, Γ is the decay rate of the inversion, and $\gamma_{\rm ph} = (2a - b)/\tau$ is the rate of phase diffusion of the drive light [40]. The factor f_i takes into account the Zeeman splitting in the ground state ($f_0 = 1/2$), and preparation in the metastable state with possibly mixed orientation ($f_1 < 1$). From the *recorded* spectrum, corresponding to the calculated one shown in Figure 3, the nutation phase $\theta = 2\pi n + \theta'$ was derived [25], where n was found about 640, and θ' close to π for peak values in the spectrum, and close to zero in the dips, corresponding to maximum and minimum probability of excitation. From the trajectories of results, the numbers of sequences have been extracted that are made up of q consecutive equal results, $U(q)$. These quantities, normalized by $U(1)$, have been compared with the joint probability $V(q)$, (Fig. 4). One of the trajectories is required for determining, from the "on" sequences, the parameter of total relaxation, $a + b$. The "on" sequences in all the remaining trajectories allow the measurement of the fractional phase of nutation, θ'. Note that this phase is available with substantial precision (see Fig. 4). The "off" sequences yield f_1. In such sequences of "no count" observations, the factor f_1 decreases from unity ($\theta' \simeq 0$) upon increasing deexcitation (growing θ'), since more and more cycles of spontaneous decay followed by stimulated reexcitation contribute to the "off" results.

The agreement of $V(q)$ and the normalized $U(q)$ reveals the driven evolution of the ion being set back during the action of each of the probe pulses. This behaviour may or may not be interpreted by repeated "reduction" to the initial energy eigenstate, or "collapse" [9] (see Ref. [8] for a comprehensive discussion of the relevance of collapse to QZE). However, there is no way to invoke dephasing, since the spin-equivalent individual quantum object – the driven quadrupole – pertains to a pure ion state and displays a well-defined phase except in the two eigenstates of energy which are, however, generated only upon particular excitation, namely by π or 2π pulses of the drive light. Thus it seems that finally the effect of measurement has been unequivocally unveiled from the disguise as physical phase perturbation that it assumes when a quantum *ensemble* is observed.

7 Back-action on the quantum object

An important issue concerning potential phase perturbation requires attention: the amount of back action of the light fields upon the individual quantum system in the course of the above procedure of measurement. A sufficient – although not a necessary – condition for the exclusion of large enough back action is the measurements being of QND type. An observation qualifying for this category needs a quantum object whose state has been entangled with a quantum probe that is subjected to the physical detection [21]. The result of this detection permits one to infer the state of the quantum object thanks to its correlation with the state of the quantum probe. A sufficient condition for QND is

$$\mathbf{U}^+ \, \mathbf{x} \, \mathbf{U} - \mathbf{x} \; = \; 0 \; , \tag{3}$$

where $\mathbf{x}$ is the operator of the relevant observable, and $\mathbf{U}$ is the operator of the joint time evolution of probe and object. This condition demands that both quantum object and quantum probe return to their respective states after the measurement. In the above scheme of observation, the ionic quadrupole induced by the drive is the quantum object, while the dipole on the resonance line is the quantum probe. Let us scrutinize the possible outcomes of the outlined procedure of measurement.

There are two kinds of results that are characterized by probe-light scattering "on" or "off". "Off" detections are unrelated with any light scattering. Moreover, they do not cause any physical recoil at all on the quantum system since the probe-line dipole and the concomitant resonance scattering remain unexcited and establish a "negative result". Note that this *absence* of light scattering allows one, thanks to the entanglement with the quantum object, to infer upon the state of the quantum object, *i.e.* the ion resting in the dark $D_{5/2}$ level.

"On" results indicate interaction of ion and probe light, by the latter inducing the oscillating dipole of the former on the probe resonance that gives rise to the scattering. Since both quantum object and probe return to their initial states after each cycle of measurement, the QND condition seems to hold. But probe light scattering also gives rise to stochastic momentum transfer to the ion. However, laser cooling establishes a stationary vibrational state; and the temporal distribution of the ion in phase space remains invariant, in agreement with the QND condition.

Now we should examine whether or not "direct interaction" [8], of the probe light with the quantum object, may effect *physical* intervention in the latter to be expressed as collapse, state reduction, or their equivalent. Such an interaction might take place *via* either 1. electronic excitation, or 2. light recoil.

1. According to the quantum system's interaction with the drive light, the superposition state $\alpha\langle n,1| + \sqrt{1-\alpha^2}\langle n+1,0|$ of the quantum object, with $\alpha = \alpha(\Omega\tau)$, is prepared and said to turn, by help of the probe light, into $\langle n+1,0|$, where n is close to the mean photon number of the coherent drive light. After the drive light has been switched off, and the probe light on, $\langle 0|$ correlates with probe light scattering "on", and $\langle 1|$ with "off", such that rather $\langle \text{"on"}, n+1, 0|$ results. But how does the probe light manage to "reduce" the quantum system into state $\langle \text{"on"}, 0|$?

For an answer, we should distinguish (i) what we *infer* from the quantum-mechanical model designed for the description of an ensemble: the preparation in the superposition state – and (ii) what we *measure* and *know* from the result of the measurement, and to what we attribute "reality": the system found again and again in the $|0\rangle$ state. This distinction is indispensable when dealing with single measurements on an individual system. If we ignored this distinction and attributed reality to the predictions of quantum mechanics, when applied to individual measurement on an individual quantum system, for the time intervals between measurements, *e.g.*, to the expectation value of energy, then both the excitation of the dressed state, and the presumed subsequent jump from the inferred left-over superposition state to $|\text{"on"}, 0\rangle$ were identifiable *physical processes.* They were going on in the quantum object, induced by interaction with drive and probe, respectively. This two-step scenario is impossible to take place in an individual atom, for sake of the atom's quantized internal energy. Moreover, the jump had to be represented by a Hamiltonian that enters the deterministic Schrödinger equation, although it is well established that the nonlinear and stochastic collapse of the wave function cannot be described by a linear, deterministic Hamiltonian interaction of system and probe. In fact, any measurement following coherent excitation results, with respective probability, in one of the *eigen*values of states 0 or 1. In a sequence of "on" observations, we know *a posteriori* that *all* the results have been "state 0": consequently, there is no motive for attributing other states to the quantum object (the driven resonance), between two measurements, and equation (3) holds. Thus, the quantum object remains free of reaction, and it is not reasonable to invoke, for sequences of equal results, "direct interaction" with the probe.

On the other hand, phase and amplitude noise that is imposed, by the probe-light pulses, upon the ion's ground-state wave function makes the quadrupole on the driven transition (the quantum object) *decohere*, although this effect cannot give rise to the above transitions.

In fact, we know from the series of QND measurements that the system keeps being found in $|\text{"on"}, 0\rangle$, and *assuming* the system to have been elsewhere between measurements is not substantiated, although the *a priori* probability for the experimental finding has been less than unity, according to preparation by the drive light. A measurement of the state acquired by this preparation in a base of which that state is an eigenstate would verify the result of preparation. If such a measurement were sandwiched in between the drive and probe pulses, it would be left, by the subsequent probing (measurement of energy state with result $|0\rangle$), an *incompatible* measurement of a non-commuting variable. The results of such two incompatible measurements cannot *simultaneously* claim reality, a quality that depends on the selected base being adapted to the observable measured. Consequently, there is no *physical* collapse that would require "direct interaction", but rather reduction of possibilities, *i.e.* enhancement of knowledge, by the measurement.

2. Scattering of the probe light may exert recoil to the quantum system However, if the vibrational frequency ν_v of the ion in the trapping potential exceeded the widths of *both* lines, the macroscopic trap would absorb the recoil exchanged in the course of any radiative interaction of ion and field, since this interaction extends over many vibrational periods [41]. In the actual experiment, this "strong trapping" holds with the drive light, although it does not hold with the probe. At any rate, the ion remains laser-cooled, while probed, and characterized by a *narrow* stationary distribution of its momentum. Detection of individual acts of recoil could, in principle, replace the detection of the scattered probe light, and so far this detection would amount to another way of measuring the state "on". As well as the recording of scattered light, this detection of momentum transfer cannot *physically* do anything like changing the quantum object's superposition state into $\langle\text{"on"}, 0|$. However, it will cause decoherence as a consequence of Doppler phase modulation of the quantum object's wave function. Since the ion is laser-cooled to an extent that leaves its vibrational excursion far smaller than half the wavelength of the light (it remains in the "Lamb-Dicke regime"), the extent of this decoherence is rather limited. The phase perturbation imposed on its wave function by the momentum transferred from the probe light is restricted to just a negligible fraction of π. Consequently, this small phase perturbation of the quantum object does not suffice for mimicking the effect of information-enhancing measurements, as a consequence of the non-local correlation of the quantum system with the result of counting, on the macroscopic level, the photoelectrons released by the scattered probe light. By the same token, substantial decoherence *via* phase-fluctuating signal light cannot be tolerated [25].

In summary, the actual conditions of the recent experiment, for both "on" and "off" results, in fact exclude physical intervention with the quantum object – the ion's quadrupole moment on the signal line – as the origin of the observed statistics of results.

8 Exponential decay vs. coherent evolution

Some objections have centered at questioning the relevance of a test of the *coherent* evolution of a quantum system as opposed to a test of the spontaneous decay of such a system [24]. Indeed, the initial concept of the retarded evolution identified with the quantum Zeno effect was concerned with spontaneous decay of a quantum system [3–5]. As discussed above, the argument for retardation to show up rests on the presumed existence of a short initial time regime when the occupation density of the decaying state decreases as the square of time, and specifically as a Gaussian. Such a regime has been shown to emerge from an unstable quantum system under very general conditions [3,4]: the non-decay probability for a quantum system decaying into a stable final state cannot, for an initial short time regime, follow an exponential which is the signature of classical evolution [30]. Thus, the strange and possibly paradoxical aspects of QZE are constituents of *any* physically unperturbed quantum evolution subjected to intervention by observations, in particular of a quantum evolution where the existence of that quadratic time dependence is obvious, as the coherent evolution. Nonetheless, testing spontaneous decay for that quadratic regime of evolution – and the concomitant QZE – could provide most valuable insight into the emergence of classical behaviour in a compound of quantum systems. Recent proposals for testing decay processes that even predict accelerated decay [33,34] are based on interaction of the quantum system with a radiative reservoir, modelled in the approximation of Fermi's golden rule. These approaches would correspond to a definition of QZE that does not discriminate between physical back action and non-local correlation [8]. However, a test of spontaneous decay for initial deviation from exponential behaviour would possibly allow one to check quantum models that differ with respect to the reservoir, *e.g.* Breit-Wigner theory [42] and models like the one detailed in reference [7]. Desirable as such a test on a spontaneously decaying quantum system seems – it is evident that the uncovering of the effects of such a transient regime requires either experimentation on an extremely short time scale, and/or with excessively high sensitivity. Therefore, it is not surprising that attempts have been surfaced in order to first manipulate the exponential decay of the quantum system such as to make it display a quadratic regime of decay that extends over a time interval more accessible by current experimental technique [24]. In order to follow this strategy, a supplementary coherent excitation is proposed to be applied to the quantum system, and a nonlinear contribution to the interaction may be detected that mixes the amplitudes of the coherent and dissipative parts. The evolution of this nonlinear contribution is supposed to display, on an accessible intermediate time scale, a t^2 evolution, or even some of the oscillatory variation that goes along with the coherent interaction but is, at later times, overwhelmed by the incoherent decay. However, this t^2 regime is the signature of just the admixed coherent contribution to the interaction, and it is by no means evident how to infer, from its observation, the existence of a much shorter t^2 regime in the *incoherent* interaction, let alone the physical origin of this transient dynamics. Thus, the strategy of reference [24] is of questionable value for the decision whether or not natural decay is accompanied by the deviation from exponential decay being the condition of QZE. For the substantial relevance the answer to this open question has upon the problem of the boundary between quantum micro-system and classical macro-system, we suggest a possible pathway out of the dilemma posed by the enormous sensitivity of detection required, as it seems, for a meaningful test of the dynamics of a "natural" exponential decay.

9 A sensitive test of exponential decay being modified by measurements

The decay time related to a resonance as considered in the above experiment [25] is $\Gamma^{-1} = \tau/2b$, and b is the quantity to be measured with utmost sensitivity in order to reveal any variation of Γ under more frequent probing. A straight-forward approach would consist of increasing the repetition rate of the drive and probe cycles, and check the corresponding trajectories of measurements for a *variation* of b derived from precise determinations of the fractional phase of nutation, θ'. Although this approach seems feasible in principle, it is yet unpractical: in fact, one natural mode of relaxation, characterized by $a+b$, is easily accessible from fitting $U(q)/U(1)$, the other one, $a-b$, is not, since it must be derived from a small difference of the large quantities θ^2 and $(\Omega\tau)^2$. A better approach would rely on the comparison of *correlated* fractional phases θ', as in interferometry, such that their relative variation is directly traced back to a variation of the relaxation constant b. This strategy is outlined in what follows.

The accumulation of trajectories of measurements may be modified such that the probe light is alternated only after two, or three, or n pulses of the drive. The corresponding phases of nutation are

$$\theta_n = \sqrt{n^2(\Omega\tau)^2 - (a-b_n)^2} = 2\pi ns + \theta'_n, \qquad (4)$$

where s is the number of integer nutational rotations induced by one driving pulse. The anticipated retardation of the quantum evolution would make b_n decrease upon decreasing n. With $\Omega\tau > \pi$, the square root is expanded in orders of $(a-b)/n\Omega\tau$. The fractional phases θ'_n may be determined with less than 10^{-4} error, and one derives, with $b_m = b_n - \delta b$,

$$\delta_{mn} \equiv \frac{\theta'_m}{m} - \frac{\theta'_n}{n} = \frac{1}{\Omega\tau}\left(\frac{a-b_n}{n}\right)^2 \times \left[1 - \left(\frac{n}{m}\right)^2\left(1 + \frac{\delta b}{a-b_n}\right)^2\right]. \qquad (5)$$

From the value δ_{m1} derived from θ'_m and θ'_n of trajectories at $n=1$ and $1 \ll m < \Gamma^{-1}/\tau$, one derives $a-b_1$, with $\Omega\tau$ taken from fitting a spectrum of the excitation, or from calibration of the light flux .

A measured value of δ_{21}, at $n=1$ and $m=2$, allows us to determine the presumed variation δb, by effectively doubling the driving period. This may be illustrated by a numerical example: with the realistic values $a \simeq 0.4$, $b_1 \simeq 0.2$, $\Omega\tau \simeq 2\pi$, we have $\delta_{m1} \simeq 6\times10^{-3} \pm 2\times10^{-4}$, and $\delta_{21} = \delta_{m1}(1-(1+5\delta b)^2/4) \simeq \delta_{m1}(3/4-10\delta b/4)$. Thus, the maximum error of δ_{21}/δ_{m1} is 4×10^{-4}, and the error of δb would amount to 1.6×10^{-4}. This level determines the limit of sensitivity for the observation of modified *decay* with the experiment described above [25].

The outlined strategy represents a kind of "heterodyne" measurement of the effective fractional phase of nutation. As a "local oscillator", the corresponding phase of the coherent evolution is made use of.

For an experimental demonstration of any observation-induced variation of the rate b of energy relaxation, the excited state of the driven resonance must live long enough to exceed a rather long sequence of, say, ten of the standard drive/probe periods, and short enough to allow the detection of the reduced decay parameter b. The $S_{1/2}$–$D_{5/2}$ transition of $^{172}Yb^+$ with its lifetime of the $D_{5/2}$ level on the order of 5 ms [43] may in fact offer a good compromise. An experiment along these lines would yield at least an upper boundary for the measurement-induced retardation (or even acceleration) of the exponential decay, although this effect may be still several orders of magnitude inferior.

10 Conclusions

The conditions for demonstrating the inhibition or retardation of the quantum evolution of an atomic system by reiterated measurements have been reviewed in the light of various arguments that had been raised with respect to an attempted demonstration some ten years ago [10], and on account of a recent experiment on an individual atom [25].

Questions about the general applicability of von Neumann's principle of state reduction already have been clarified when a single-particle model calculation proved this principle to provide an excellent approximation to the ideal evolution of the system [9]. Moreover, the existence or nonexistence of state reduction and collapse have been shown irrelevant to the problem of the measurement-affected evolution [8].

In order to identify the retarding effect of observation by a quantitative evaluation, it seems indispensable to *register* the results of all the interrogations. Otherwise, correlated up-down transitions in pairs of atoms, or back-and-forth transitions in a single atom, would falsify the crucial probability for no transition that is derived from the observations at least in a real experiment [17].

The most serious objections have been aimed on the very nature of the impeding effect. It has been argued that, for a signature, the measurements should be nonlocal and of the negative-result type [8]. Although we acknowledge this requirement as a sufficient condition, there is good reason to accept, as "good" measurements, QND measurements with positive results, and non-QND measurements as long as the back action *demonstrably* cannot cause the inhibition of evolution.

The impeding effect of *measurement* has been, in the meantime, proven indistinguishable from dephasing the system's wave function, *except* for an individual particle [18–23]. An experiment on such a system, a single trapped and laser-cooled ion, including the read-out of the results of *all* interactions with the probe light, has been reported recently [25]. The results of this experiment show that the observed retardation of the system's evolution is unequivocally traced to the repeated interrogation of the quantum system's internal state of energy, and *not* to dephasing of the system's wave function.

The controversy also dwelled upon the experimental test addressing *coherent* evolution as opposed to exponential decay [24]. In fact, unstable quantum systems are supposed to include an initial very short regime of quadratic time evolution that is prerequisite also of the "true" Zeno variant of quantum-evolutive retardation. If a system lacked this regime, this finding would dramatically highlight the discrepancy between a quantum micro-system, and such a classical macro-system emerging from the action of a reservoir. As for a proof, the admixture of a coherent contribution to the system's evolution as previously suggested [24] is of questionable evidential value. On the other hand, the anticipated time delay of an exponential decay may be determined, with substantial precision, by a modification of the reported drive-probe experiment. At least a meaningful upper bound for that elusive retardation might be hoped for.

The evolution of a quantum system is always predicted, with the help of the deterministic Schrödinger equation, on the base of previous measurements or assumptions, that determine the initial conditions. Even the *availability* of additional information on the system requires partial updating, on the ground of a potential measurement having happened. The retardation of the quantum evolution is the signature of this continuated process of updating. The predictable quantum evolution conditions the stochastic "factual" evolution of the system, at least in the probabilistic interpretation of quantum mechanics. If this is so, the Quantum Zeno Effect emerges as a consequence of all, that we can know about a quantum system's evolution, remaining incomplete.

References

1. J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Springer Verlag, Berlin, 1932).
2. G. Lüders, Ann. Phys. (Lpg) **8**, 323 (1951).
3. L.A. Khalfin, Pis'ma Zh. Eksp. Teor. Fiz. **8**, 106 (1968) [JETP Lett. **8**, 65 (1968)].
4. L. Fonda, G.C. Ghirardi, A. Rimini, T. Weber, Nuovo Cimento A **15**, 689 (1973).
5. B. Misra, E.C.G. Sudarshan, J. Math. Phys. (N.Y.) **18**, 756 (1977).
6. A. Peres, Am. J. Phys. **48**, 931 (1980).
7. H. Nakazato, M. Namiki, S. Pascazio, Int. J. Mod. Phys. B **10**, 247 (1996).
8. D. Home, M.A.B. Whitaker, Ann. Phys. (N.Y.) **258**, 237 (1997).
9. A. Beige, G.C. Hegerfeldt, Phys. Rev. A **53**, 53 (1996).
10. W.M. Itano, D.J. Heinzen, J.J. Bollinger, D.J. Wineland, Phys. Rev. A **41**, 2295 (1990); *ibid.* **43**, 5168 (1991).
11. P. Kwiat, H. Weinfurter, T. Herzog, A. Zeilinger, M.A. Kasevich, Phys. Rev. Lett. **74**, 4763 (1995).
12. L.E. Ballentine, Phys. Rev. A **43**, 5165 (1991).
13. V. Frerichs, A. Schenzle, Phys. Rev. A **44**, 1962 (1991).
14. T. Petrosky, S. Tasaki, I. Prigogine, Phys. Lett. A **151**, 109 (1990); Physica A **170**, 306 (1991).
15. E. Block, P.R. Berman, Phys. Rev. A **44**, 1466 (1991).
16. A. Fearn, W.E. Lamb, Phys. Rev. A **46**, 1199 (1992).
17. H. Nakazato, M. Namiki, S. Pascazio, H. Rauch, Phys. Lett. A **217**, 203 (1996).
18. G.J. Milburn, J. Opt. Soc. Am. B **5**, 1317 (1988).
19. A. Peres, A. Ron, Phys. Rev. A **42**, 5720 (1990).
20. T.F. Jordan, E.C.G. Sudarshan, P. Valanju, Phys. Rev. A **44**, 3340 (1991).
21. V.B. Braginsky, F.Ya. Khalili, *Quantum Measurement* (Cambridge University Press, Cambridge, MA, 1992).
22. T.P. Spiller, Phys. Lett. A **192**, 163 (1994).
23. O. Alter, Y. Yamamoto, Phys. Rev. A **55**, R2499 (1997).
24. M.B. Plenio, P.L. Knight, R.C. Thomson, Opt. Commun. **123**, 278 (1996).
25. Chr. Balzer, R. Huesmann, W. Neuhauser, P.E. Toschek, Opt. Commun. **180**, 115 (2000).
26. G.C. Hegerfeldt, T.S. Wilser, in *Classical and Quantum Systems. Proceedings of the 11th. International Wigner Symposium 1991*, edited by H.D. Doebner, W. Scherer, F. Schroeck (World Scientific, Singapore, 1992), p. 104; G.C. Hegerfeld, Phys. Rev. A **47**, 449 (1993).

27. J. Dalibard, Y. Castin, K. Moelmer, Phys. Lett. **68**, 580 (1992).
28. M.J. Gagen, G.J. Milburn, Phys. Rev. A **47**, 1467 (1993).
29. R. Cook, Phys. Scripta T **21**, 49 (1988).
30. L. Fonda, G.C. Ghirardi, A. Rimini, Rep. Prog. Phys. **41**, 587 (1978).
31. R.G. Winter, Phys. Rev. **123**, 1503 (1961).
32. V. Weisskopf, E. Wigner, Z. Phys. **63**, 54 (1930); *ibid.* **65**, 18 (1930).
33. A.G. Kofman, G. Kurizki, LANL e-print `quant-ph/9912077` (1999).
34. M. Lewenstein, K. Rzazewski, Phys. Rev. A **61**, 022105 (2000).
35. R.P. Feynman, F.L. Vernon, R.W. Hellwarth, J. Appl. Phys. **28**, 49 (1957).
36. R. Huesmann, Ch. Balzer, Ph. Courteille, W. Neuhauser, P.E. Toschek, Phys. Rev. Lett. **82**, 1611 (1999).
37. W. Nagourney, J. Sandberg, H. Dehmelt, Phys. Rev. Lett. **56**, 2797 (1986).
38. Th. Sauter, W. Neuhauser, R. Blatt, P.E. Toschek, Phys. Rev. Lett. **57**, 1696 (1986).
39. H.C. Torrey, Phys. Rev. **76**, 1059 (1949).
40. M. Sargent III, M.O. Scully, W.E. Lamb Jr, *Laser Physics* (Addison-Wesley Publishing Co., Reading, MA, 1974).
41. S. Stenholm, Rev. Mod. Phys. **58**, 699 (1986).
42. G. Breit, E.P. Wigner, Phys. Rev. **49,** 519 (1936).
43. B.C. Fawcett, M. Wilson, At. Nucl. Data Tables **47**, 241 (1991).